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Lumped parameter modelling and an introduction to optimization of one-dimensional nonadiabatic drying systems

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Abstract—A lumped parameter model, solved numerically, is proposed for drying in several one-dimensional systems. The model appears to be an advanced extension of the standard drying description based on the drying coefficient. This extension takes into consideration the solid enthalpy changes, partial transfer resistances of heat and mass in gas and solid phases and variation of all interface parameters. A unified description of the drying kinetics is given which can be applied to all periods of drying and which clarifies, among others, the establishment of the wet bulb state. Some attention is given to the kinetics of drying with a moving evaporation front. Optimization models of several dryers (cocurrent, countercurrent and crosscurrent type) are derived on the basis of kinetic and balance equations. The models comprise differential or difference constraints, i.e. state equations, algebraic constraints as well as a performance index (optimization criterion) formulated as the drying time or the total economic cost. The minimum drying time problem is solved for the process in a horizontal fluidized exchanger and a nontrivial nature of the optimal policy is shown for low final temperatures of a solid. Other optimization results found for the models of the remaining processes, will be reported.

1. INTRODUCTION

THE FOLLOWING essential features of any drying process should be taken into account when describing rigorously the physical phenomena in any dryer: (a) equations of heat and mass transport are coupled and hence isothermal kinetics models are often insufficient, (b) equilibrium parameters at the interface vary and their instantaneous values should be found from an interface balance, (c) knowledge of the thermodynamical relationships for solid and gas is essential. However, if distributed mathematical models [1, 2] are used all these features make the solving of partial differential equations of drying extremely difficult, especially when gas conditions vary, which is, in fact, usual in optimization [3]. If the features mentioned above are not inserted into mathematical expressions describing drying kinetics then so large a simplification of the physical situation is obtained that the engineering value of results becomes questionable. Therefore, for an optimization, a general lumped parameter model (comprising the above-mentioned basic features of drying) is here proposed, although this leads to other simplications resulting from operating with the averaged values of solid temperatures and moisture concentrations rather than with the corresponding fields of these quantities.

It should be noted that the lumped parameter description may seem exceptional in the drying technique. In fact, the most popular approach [2, 4] based on the so-called drying coefficient using the well-known equation

$$\frac{dW_s}{d\tau} = K_s(W_g - W_s) \qquad (1)$$

is also of a lumped parameter type. This equation deals with averaged solid moisture content and it is commonly applied to describe drying kinetics during the so-called second drying period. However, since K_s is an overall coefficient, the complicated nonisothermal phenomena at the interface cannot be described with sufficient accuracy by any method using equation (1). A similar remark is valid for the overall coefficient K'_g associated with the driving force $X_g - X_s$. This is why the experiments [5] prove that the overall drying coefficients, K'_s and K'_g, are of restricted applicability in highly nonisothermal drying, i.e. in those cases when the cold moist solid contacts with a hot gas or when the warm solid contacts with a cold gas. In such solutions considerations related to the solid enthalpy changes as well as the use of partial transfer coefficients for both solid and gaseous phases turn out to be quite sufficient for appropriate accuracy of the lumped parameter description of drying. The corresponding method is outlined in Section 2. The method may be applied equally well when the solid is at the first or at the second period of drying. In fact, all possible phases of drying can be described in a uniform manner. For example, for constant drying conditions the method takes into account transient cooling (or heating) of solid in the course of which the wet bulb state is attained, the first drying period when $t_s = t_{wet}$ as well as the complex heating of the moist material in the second drying period.† The virtue of the uniformity of the method (rather exceptional in comparison to other approaches) results from there being no assumptions imposed on the

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† This period begins when the hygroscopic state at the interface is attained.

NOMENCLATURE

A	apparatus cross-sectional area [m^2]
a_s	area of geometrical surface per unit mass of dry solid [$\mathrm{m}^2\ \mathrm{kg}^{-1}$]
a_v	heating or cooling surface area per unit volume of dryer [$\mathrm{m}^2\ \mathrm{m}^{-3}$]
Bi_q, Bi_m	Biot's number for heat and mass exchange, respectively [dimensionless]
C_h	coefficient characterizing kinetics of external heating or cooling, $v_s\alpha_h a_v/g'$ [$\mathrm{kJ\ kg}^{-1}\ \mathrm{K}^{-1}$]
C_g, C_s	heat capacity of moist gaseous or solid phase per unit mass of dry gas or dry solid, respectively [$\mathrm{kJ\ kg}^{-1}\ \mathrm{K}^{-1}$]
c_g, c_w	heat capacity of dry gas and moisture, respectively [$\mathrm{kJ\ kg}^{-1}\ \mathrm{K}^{-1}$]
D_{ef}	effective diffusivity of moisture vapour in boundary region [$\mathrm{m}^2\ \mathrm{s}^{-1}$]
$f(t_g X_g, I_s, W_s)$	kinetical function describing $\mathrm{d}I_s/\mathrm{d}W_s$ in terms of t_g, X_g, I_s and W_s [$\mathrm{kJ\ kg}^{-1}$]
$\tilde{f}(i_g, X_g, I_s, W_s)$	kinetical function describing $\mathrm{d}I_s/\mathrm{d}W_s$ in terms of i_g, X_g, I_s and W_s [$\mathrm{kJ\ kg}^{-1}$]
G	dry gas flow rate [$\mathrm{kg\ s}^{-1}$]
$g(t_g, X_g, I_s, W_s)$	kinetical function describing $\mathrm{d}\tau/\mathrm{d}W_s$ in terms of t_g, X_g, I_s and W_s [s]
$\tilde{g}(i_g, X_g, I_s, W_s)$	kinetical function describing $\mathrm{d}\tau/\mathrm{d}W_s$ in terms of i_g, X_g, I_s and W_s [s]
g'	gas flow per unit residence time of solid in crosscurrent system, $\mathrm{d}G/\mathrm{d}\tau$ [$\mathrm{kg\ s}^{-2}$]
H	Hamiltonian (Pontryagin's function) [s]
I_b, I_g, I_i	averaged solid enthalpies; at geometrical surface, for solid in equilibrium with gas and at interface (Fig. 1), respectively [$\mathrm{kJ\ kg}^{-1}$]
$I_s, \bar{I}_s$	averaged enthalpy of solid phase per unit mass of absolute dry material for moist solid and total solid, respectively [$\mathrm{kJ\ kg}^{-1}$]
I_s^n, I_s^{n-1}	enthalpy of solid from stage n and before stage n, respectively [$\mathrm{kJ\ kg}^{-1}$]
I_{s0}, I_{sf}	initial and final solid enthalpy, respectively [$\mathrm{kJ\ kg}^{-1}$]
i_b, i_g, i_i, i_s	gas enthalpies (per unit mass of absolute dry gas); at geometrical surface, for gas phase, at interface and for gas in equilibrium with most solid (Fig. 1), respectively [$\mathrm{kJ\ kg}^{-1}$]
i_e	enthalpy of external gas [$\mathrm{kJ\ kg}^{-1}$]
i_v, i_l	partial enthalpy of moisture in vapour and liquid state, respectively [$\mathrm{kJ\ kg}^{-1}$]
i_m	two-phase stream enthalpy per unit mass of dry solid, $\theta i_g + I_s$ [$\mathrm{kJ\ kg}^{-1}$]
J	total economic value of dryer [\$]

K	total unit cost of the process defined as a sum of unit investment and operational costs, $K^i + K^e$ [\$ kg^{-1}]
K^i, K^e	investment and operational costs referred to the unit flow of dry solid, respectively [\$ kg^{-1}]
K'_g, k'_g	modified overall and partial mass transfer coefficient, associated with driving forces $X_s - X_g$ and $X_b - X_g$, respectively [$\mathrm{kg\ m}^{-2}\ \mathrm{s}^{-1}$]
k'_{gb}	modified gas mass transfer coefficient comprising gas boundary layer and solid boundary region, associated with driving force $X_i - X_g$ [$\mathrm{kg\ m}^{-2}\ \mathrm{s}^{-1}$]
K_s, k_s	overall and partial drying coefficient, respectively [s^{-1}]
K'_s, k'_s	modified overall and partial drying coefficients, associated with driving force $W_s - W_g$ and $W_s - W_i$, respectively [$\mathrm{kg\ m}^{-2}\ \mathrm{s}^{-1}$]
Le, Lu	Levis factor and Luikov factor, equations (8) and (9), respectively
M_g	molecular mass of dry gas phase [$\mathrm{kg\ kmol}^{-1}$]
$\dot{M}_j, \dot{M}_w$	mass flow of jth component and moisture, respectively [$\mathrm{kg\ s}^{-1}$]
m_g	external mass flow of moisture added per unit gas flow, $\dot{M}_w/G$ [$\mathrm{kg\ kg}^{-1}$]
N, n	total number of stages and current stage number, respectively
P, P_s	total pressure and saturated vapour pressure, respectively [$\mathrm{N\ m}^{-2}$]
Q	total external heat flow [kJ]
q_g	external heat per unit mass of gas [$\mathrm{kJ\ kg}^{-1}$]
R	universal gas constant [$\mathrm{kJ\ kmol}^{-1}\ \mathrm{K}^{-1}$]
R_0	radius of spherical particle, characteristic dimension [m]
r_0	evaporation heat of liquid moisture at $T = 273.2$ K [$\mathrm{kJ\ kg}^{-1}$]
S	dry solid flow rate [$\mathrm{kg\ h}^{-1}$]
S_0	dry solid mass in apparatus [kg]
T	absolute temperature of a particular phase [K]
T_u	utilization time of dryer [$\mathrm{h\ year}^{-1}$]
T_{gr}	maximum acceptable payout time [years]
t_b, t_g, t_i, t_s	averaged temperatures; at geometrical surface, of gas phase, at interface and of solid phase, respectively [K]
t_e, t_h	temperature of external gas and of external medium controlling the process, respectively [K]
t_*, t^*	minimal and maximal allowable temperature, respectively [K]

v_s	solid velocity, positive for cocurrent flow and negative for countercurrent flow, respectively [m s^{-1}]
W_b, W_g, W_i	averaged moisture content in solid; at geometrical surface, for solid in equilibrium with gas and at interface (Fig. 1), respectively [kg kg^{-1}]
$W_s, \bar{W}_s$	averaged moisture content in solid phase per unit mass of absolute dry substance, for moist solid and total solid, respectively [kg kg^{-1}]
W_s^n, W_s^{n-1}	moisture content in solid from stage *n* and before stage *n*, respectively [kg kg^{-1}]
W_{s0}, W_{sf}	initial and final solid moisture content, respectively [kg kg^{-1}]
X_b, X_g, X_i, X_s	gas humidities per unit gas of absolute dry gas; at geometrical surface, for gas phase, at interface and for gas in equilibrium with moist solid (Fig. 1), respectively [kg kg^{-1}]
X_e	humidity of external gas [kg kg^{-1}]
X_m	two phase stream moisture content, $\theta X_g + W_s$ [kg kg^{-1}]
$\tilde{z}$	factor describing 'freezing' of capital costs, equation (58)
z	adjoint variable [s kg kJ^{-1}].

Greek symbols

α_g, α_s	partial coefficient of heat transfer for gaseous and solid phase, respectively [kJ m^{-2} s^{-1} K^{-1}]
α_{gb}	heat transfer coefficient comprising gas boundary layer and solid boundary region, associated with driving force $t_i - t_g$ [kJ m^{-2} s^{-1} K^{-1}]
α_h	coefficient of heat exchange between an external heating (or cooling) medium and gas [kJ m^{-2} s^{-1} K^{-1}]
$\beta_i, \beta_g \beta_g^*$	relative humidity of gas at the interface, gas relative humidity and maximum allowable value of β_g, respectively [dimensionless]
$\tilde{\beta}$	coefficient describing repairs [$year^{-1}$]
Δ	increment
δ	depth of boundary region [m]
θ	gas–solid ratio, equation (26)
λ_{ef}	effective thermal conductivity [kJ m^{-1} s^{-1} K^{-1}]
ρ	density [kg m^{-3}]
τ	solid residence time, drying time [s]
$\bar{\tau}$	time constant, S_0/G [s]
ϕ	dimensionless residence time of solid for a crosscurrent dryer, $G\tau/S_0 = G/S$.

Subscripts

b	geometrical surface
e	external gas
f	final value
g	gas bulk
h	heating (or cooling) medium
i	interface
l	liquid
m	mixed gas–solid stream
s	solid (core)
w	moisture
0	boundary value
1	end or switching value.

Superscripts

e	exploitation cost
i	investment cost
n	stage *n*
v	prescribed value
′	new state
0	initial state
1	stage 1.

interface parameters, i.e. isothermicity assumption, made tacitly in many other approaches, is here omitted. This allows for a precise use of the complex interphase equilibria $X_i(t_i, W_i)$, $i_i(t_i, W_i)$ etc. Furthermore, the dependence of any coefficient on solid residence time can be taken into account in a computer program owing to the numerical nature of the method.

The project of this investigation is as follows. Firstly, the already mentioned model describing drying kinetics, equations (10)–(12), is derived in Section 2. Its applicability to arbitrary dryers with variable gas path is stressed and a graphical interpretation of the numerical method of model solving is presented in Section 3 where some attention is also given to the kinetics of drying with a moving evaporation front. Secondly, the algebraic constraints and the so-called state equations, i.e. differential constraints linking gas and solid parameters, are derived. This is done in Section 4 for one-dimensional nonadiabatic dryers (cocurrent, countercurrent and crosscurrent types) by combining drying kinetics equations with additional relationships specific for each type of dryer (found usually from mass and heat balance). The optimization criteria are discussed in Section 5. As the basic purpose of this paper is the modelling and the formulation of optimization problems, the specific results of optimization are discussed for the relatively simple problem of a minimum drying time only (Section 6). Other optimization results, for the remaining processes modelled here, will be reported (see also refs. [6, 7] dealing, however, with adiabatic systems exclusively).

The mode of modelling given here is especially suitable for systems with granular solids. However, the method will also work well for arbitrary dryers providing that the requirement of the one dimensionality of the dryer (variation of solid and gas states in a particular direction of space) is met. Unsteady-state dryers can also be searched for, in which the only state variations are in chronological time, e.g. fluidization dryers with ideal mixing.

2. DRYING KINETICS MODEL

When optimizing drying systems, the variation of gas state with solid residence time is required, i.e. variable drying conditions must be met [3]. Consider a solid of a definite form (sphere, cylinder, etc.) immersed in the gas changing its state in an arbitrary dryer. Figure 1 depicts a general situation when the evaporation front advances from the outer (geometrical) surface into the solid, leaving behind a region of evaporated free moisture and inert solid. The core of moist solid, S, shrinks as the evaporation front moves into the solid. Averaged parameters of region B and gas G as well as of corresponding equilibrium parameters are explained in Fig. 1.

First assume that a less general and a simple drying mechanism takes place than the one implied by Fig. 1. Namely the moisture in the solid is transferred in a liquid form exclusively and the transfer is of a diffusive nature. The interface is then identified with the geometrical surface of the solid and region B does not exist, i.e. $X_b = X_i$, $i_b = i_i$, $t_b = t_i$, etc. A more involved model for moving interface will be considered later.

In the case considered and for an isobaric process the averaged state of the solid is described by enthalpy I_s and moisture content W_s whereas the averaged bulk gas state by enthalpy i_g and humidity X_g (lumped parameter description). The following equilibrium states can be assigned to the above-mentioned quantities: the state (i_s, X_s) of gas at equilibrium with solid (I_s, W_s) and the state (I_s, W_g) of solid at equilibrium with gas (i_g, X_g). The drying kinetics can be based on the overall driving forces $W_s - W_g$ or $X_s - X_g$ and the respective overall transfer coefficients K_s' and K_g'. In a more exact description, analysed in detail below, partial driving forces and partial transfer coefficients are used and the interface parameters (i_i, X_i, I_i, W_i, t_i) are taken into consideration.

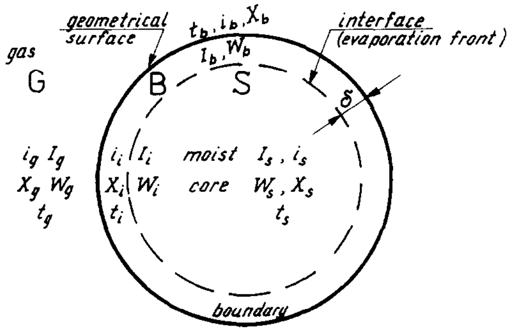

FIG. 1. The lumped parameter description of a dried sphere.

The drying rate $dW/d\tau$ is the total moisture flux multiplied by the surface area per unit mass of dry solid, a_s. Since this flux is continuous across the evaporation front the following equations hold for the stationary interface

$$\frac{dW_s}{d\tau} = k_g'(X_g - X_i)a_s = k_s'(W_i - W_s)a_s \times (X_i = X_b, \ldots, \text{etc.}). \quad (2)$$

Here $k_s' = k_s/a_s$ is the modified partial drying coefficient associated with the difference between the interface moisture content and the average moisture content of the solid. k_s' can be found experimentally or from the drying theories, especially the quasistationary or regular regime theories [1, 2, 8] of symmetric problems where k_s' is obtained as a coefficient of an asymptotic relation for the large Fourier number, Fo. As a more comprehensive consideration of this problem will be given in a separate paper only those practical results which are important for the present treatment will be presented here. It turns out that if the frequency of change of the gas parameters, ω, is not too large (i.e. Predvoditelev number, $Pd = \omega R_0^2/D_{ef}$, is small vs the square of the first root of the characteristic equation, μ_1^2, found for the constant gas state problem) then, for large Fo, the partial drying coefficients, k_s', are the well-defined constants which are independent of Biot numbers and they can be evaluated from the quasistationary regime solutions as

$$k_s' = \frac{k_s}{a_s} = \frac{\Gamma_0 \rho_s D_s}{R_0}. \quad (3)$$

where D_s is the effective moisture diffusivity in the solid and Γ_0 is the shape factor equal to 3 for an infinite plate, 4 for an infinite cylinder and 5 for a sphere. Furthermore, equation (3) gives results very close to those obtained from the regular regime theory [8] where a slight dependence of k_s' on Biot number is observed. The maximum discrepancy between k_s' found for these two regimes does not exceed 25% within the whole range of Biot numbers $0 < Bi < \infty$. In the range $0 < Bi < 1$ the discrepancy is even lower, of the order of 10%. It is also of interest that the values of the true coefficient for the process of heating of moist material before the moist thermometer state is attained lie precisely between those found for quasistationary and regular regimes. Formula (3) (and related formula (5)) is exact for all analytical solutions for which the quasistationary state can be attained, e.g. for linear time change of gas parameters, for constant surface fluxes, for stationary states with constant heat sources, etc. Thus the quasistationary regime theory results can be effectively applied, unless an alternative decision is taken to perform certain experiments [4] leading to the evaluation of k_s'. The experimental determination of the time-averaged k_s' is especially recommended when the solid residence time in the dryer is short (small Fo) which occurs, e.g. for pneumatic dryers.

Equation (2) should be considered jointly with the energy equation written in the form which takes into account the continuity of the total energy flux. This flux is equal to the sum of the sensible heat flux and the flux of enthalpy transferred with moisture. Hence

$$\frac{\mathrm{d}I_s}{\mathrm{d}\tau} = [\alpha_g(t_g - t_i) + i_{vi}k'_g(X_g - X_i)] \times a_s = [\alpha_s(t_i - t_s) + i_{li}k'_s(W_i - W_s)]a_s. \quad (4)$$

This equation relates the solid enthalpy changes, i_s, to the total energy flux as expressed in terms of gas boundary layer parameters (middle term) or solid parameters (last term). Symbols i_{vi} and i_{li} pertain to the partial enthalpies of moisture at the interface in vapour and liquid states, respectively. The heat coefficient, α_s, the thermal counterpart of the drying coefficient k'_s, is a measure of the thermal resistance of the solid. It can also be evaluated from the quasistationary regime theory as

$$\alpha_s = \frac{\Gamma_0 \lambda_s}{R_0}. \quad (5)$$

In optimization the solid is the controlled phase and the gas is the controlling phase. Hence the gas parameters (t_g, X_g) or (i_g, X_g) should be understood as time-dependent controls, $t_g(\tau)$, $X_g(\tau)$, etc., which have to extremize some performance criterion, formulated usually as an economic cost (see further). Hence the task is to solve equations (2) and (4) for the arbitrary functions $t_g(\tau)$ and $X_g(\tau)$; such solutions will define the solid trajectory $I_s(\tau)$, $W_s(\tau)$ (and $t_s(\tau)$) as well as all the interface parameters $t_i(\tau)$, $X_i(\tau)$, etc. In order to solve equations (2) and (4) the thermodynamic formulae describing solid enthalpy $I_s(t_s, W_s)$ and drying equilibrium $X_i(t_i, W_i)$ should be known. $I_s(t_s, W_s)$ is known from the specific heat data and it uses the heat of sorption of liquid moisture by dry solid [9]. Whereas $X_i(t_i, W_i)$ is evaluated directly from the drying equilibrium data, β_i as

$$X_i(t_i, W_i) = \frac{\beta_i P_s(t_i)}{P - \beta_i P_s(t_i)} \cdot \frac{M_w}{M_g}. \quad (6)$$

The specific examples of semi-empirical formulae for $\beta_i(t_i, W_i)$ and $X_i(t_i, W_i)$ are given elsewhere [9]. In the following the gas enthalpy function $i_g(t_g, X_g)$ computed in a standard manner [9] is also occasionally exploited. Its changes, or differences related to the drying forces, are approximated by the formula

$$i_g - i_i = C_g(t_g - t_i) + i_{vi}(X_g - X_i). \quad (7)$$

All the relationships discussed above allow one to solve numerically the set of equations (2) and (4). However, in the computational program it is more convenient to use W_s as an independent variable. Thus an equivalent set resulting from a simple transformation of these equations is treated numerically. For example the derivative $\mathrm{d}I_s/\mathrm{d}W_s$ is obtained by division of the energy transfer equation (4) by the mass transfer equation (2), etc. After introduction of the dimensionless factors

$$Le = \frac{k'_g C_g}{\alpha_g} \quad \text{Lewis factor} \quad (8)$$

$$Lu = \frac{k'_s C_s}{\alpha_s} \quad \text{Luikov factor} \quad (9)$$

the transformed set takes the form

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \frac{C_g(t_g - t_i)}{Le(X_g - X_i)} + i_{vi} = \frac{C_s(t_i - t_s)}{Lu(W_i - W_s)} + i_{li} \quad (10)$$

$$\frac{X_g - X_i}{W_i - W_s} = \frac{k'_s}{k'_g} \quad (11)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = \frac{1}{k'_g(X_g - X_i)a_s}. \quad (12)$$

In general, the variation of model coefficients (Le, Lu, etc.) should be taken into account.

The superiority of the set, equations (10)–(12), in relation to the original set, equations (2) and (4), relies on the fact that equations (10) and (11) can be solved independently of equation (12) and, therefore, the computational procedure is broken down to two separate integrations. In addition the transformed set is more suitable for the optimization as it allows one to reduce the problem dimensionally via a Lagrangian multiplier [3]. Finally, the solution of equations (10) and (11) has an interesting and valuable interpretation on the generalized enthalpy diagram [10] for gaseous and solid phases, Fig. 2. This solution comprises the

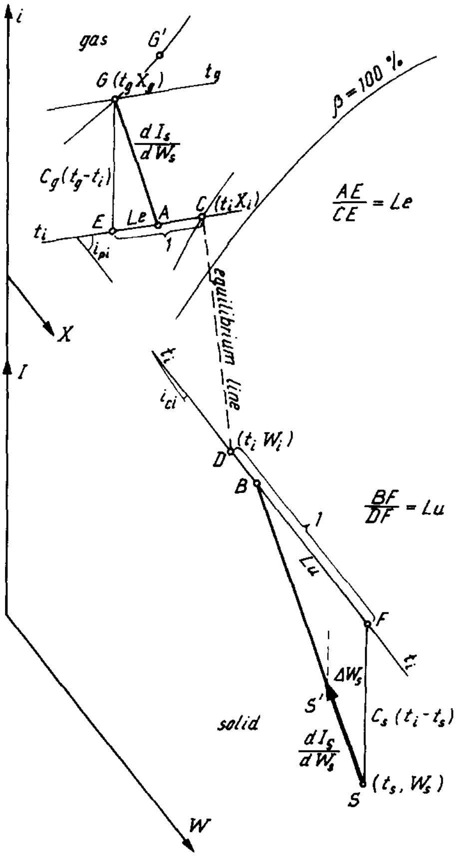

FIG. 2. Graphical interpretation of finding the derivative $\mathrm{d}I_s/\mathrm{d}W_s$ and interface parameters on the gas–solid enthalpy diagram.

function $I_s(W_s)$ (and hence $t_s(W_s)$) as well as the interface parameters t_i, X_i and W_i found locally for the definite states of gas (t_g, X_g) and solid (I_s, W_s). When the gas parameters t_g and X_g change in a definite manner with solid moisture content, e.g. when the functions $t_g(W_s)$ and $X_g(W_s)$ are preassigned, the outcome of the solution of equations (10)–(12) is the solid path, $I_s(W_s)$ or $t_s(W_s)$, and the drying time function, $\tau(W_s)$, for the given initial state of the solid (I_{s0}, W_{s0}).

3. GRAPHICAL INTERPRETATION OF THE NUMERICAL PROCEDURE

Figure 2 depicts the solution of equations (10) and (11). For a definite gas state, $G(t_g, X_g)$, and an instantaneous solid state, $S(I_s, W_s, t_s)$, the solid interface parameters (t_i, W_i) are assumed. From equilibrium data, the state of the gas at the interface (t_i, X_i) is determined, and the partial enthalpies of moisture $i_{vi}(t_i, X_i)$ and $i_{li}(t_i, W_i)$ are found. Graphically i_{vi} and i_{li} are interpreted as the directional coefficients of the isotherm t_i for gaseous and solid phases, respectively. If the state (t_i, W_i) was assumed correctly, the directional coefficients of straight lines GA and BG should be the same and equal to $\mathrm{d}I_s/\mathrm{d}W_s$ providing that the ratio $(X_i - X_g)/(W_s - W_i)$ is equal to k'_s/k'_g. Assuming $(\mathrm{d}I_s/\mathrm{d}W_s) \cong (\Delta I_s/\Delta W_s)$ and a definite small ΔW_s the new solid enthalpy is found on the straight line GA at the point $S'(I_s+\Delta I_s, W_s+\Delta W_s)$. This point defines the new solid state. When the new gas state G' is known, for the definite control, the procedure can be repeated, leading finally to the solid path $I_s(W_s)$. The corresponding drying time function $\tau(W_s)$ is then found by integration of equation (12). When the gas state is constant and the solid is in the over hygroscopic regime it results from the construction in Fig. 2 that even if the differences between the temperatures t_s and t_i are initially large they will decrease fast and an identical wet bulb temperature is attained in the whole solid, i.e. after some time $t_s = t_i = t_{\text{wet}}$. This fact is well known from the drying experiments.

The above procedure, which can be organized numerically or graphically, can be extended to the situation when the evaporation front moves toward the interior of the solid, Fig. 1. In this case [11] there exists an external boundary region in the solid, of variable depth δ (phase B, Fig. 1) in which the moisture is transferred through the gas contained in the pores of the material. As this transfer occurs in the vapour state the sensible heat flux is continuous across the geometrical surface of the solid if the changes of the vapour enthalpy of moisture i_v are neglected. Hence the overall transfer between the moving interface and the gas bulk can be approximately characterized by the following coefficients

$$\alpha_{gb} = \left(\frac{1}{\alpha_g} + \frac{\delta}{\lambda_{ef}}\right)^{-1} \tag{13}$$

$$k'_{gb} = \left(\frac{1}{k'_g} + \frac{\delta RT}{D_{ef}PM_g}\right)^{-1} \tag{14}$$

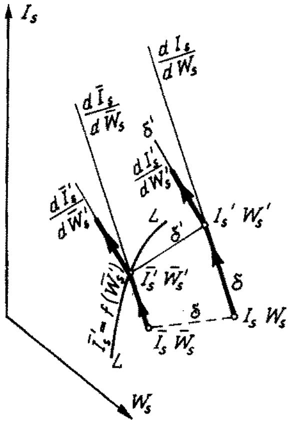

FIG. 3. Estimation of the paths $I_s(W_s)$ and $\bar{I}_s(\bar{W}_s)$ for a model of drying with a moving evaporation front.

$$Le_{gb} = \frac{C_g(1/\alpha_g + \delta/\lambda_{ef})}{(1/k'_g + \delta RT/D_{ef}PM_g)}. \tag{15}$$

Here λ_{ef} and D_{ef} are, respectively, the effective thermal conductivity and the effective diffusion coefficient of moisture in the porous envelope. Equations (13)–(15) can be applied for small δ in comparison to the sphere radius, R_0, else they should be modified by introduction of a correction factor related to the surface areas of the solid and the interface.

In the case considered the parameters I_s and W_s are related to the moist core whereas the averaged state of the solid ($\bar{I}_s$, $\bar{W}_s$) can be found from consideration of the additive contributions of the core S and boundary region B. Assuming that the average moisture content of the boundary region is $\frac{1}{2}(W_i + W_b)$ and its enthalpy is $\frac{1}{2}(I_i + I_b)$ the averaged parameters of the spherical solid $\bar{I}_s$ and $\bar{W}_s$ are

$$\bar{W}_s = \frac{1}{2}\left[1 - \left(1 - \frac{\delta}{R_0}\right)^3\right](W_i + W_b) + \left(1 - \frac{\delta}{R_0}\right)^3 W_s \tag{16}$$

$$\bar{I}_s = \frac{1}{2}\left[1 - \left(1 - \frac{\delta}{R_0}\right)^3\right](I_i + I_b) + \left(1 - \frac{\delta}{R_0}\right)^3 I_s. \tag{17}$$

The computational procedure (Fig. 3) starts from the assumption of the initial state of the core (I_s, W_s) and the depth δ. Next, the computations described earlier, leading to the interface parameters (t_i, W_i, X_i) and the derivative $\mathrm{d}I_s/\mathrm{d}W_s$, are performed, providing that the partial gas coefficients, α_g, k'_g and Le_g, are now replaced by the gas-boundary region coefficients, α_{gb}, k'_{gb} and Le_{gb}, equations (13)–(15). Then the geometrical surface parameters t_b and X_b are found from the transfer formulae

$$\alpha_g(t_b - t_g) = \frac{\lambda_{ef}}{\delta}(t_i - t_b) \tag{18}$$

$$k'_g(X_b - X_g) = \frac{D_{ef}PM_g}{\delta RT}(X_i - X_b). \tag{19}$$

When t_b and X_b are known the solid state (I_b, W_b) at the geometrical boundary is found from the equilibrium data and the average solid state ($\bar{I}_s$, $\bar{W}_s$) is computed from equations (16) and (17). Hence all parameters corresponding to the initial state of the solid (I_s, W_s, $\bar{I}_s$, $\bar{W}_s$) are known, any parameter being dependent on δ.

Now the new state of the moist core, $I'_s = I_s + \Delta I_s$ and $W'_s = W_s + \Delta W_s$, is found for the assumed small ΔW_s. As previously this state is found on the ray which has the directional coefficient $\mathrm{d}I_s/\mathrm{d}W_s$. Since

$$\frac{\mathrm{d}\bar{I}_s}{\mathrm{d}\bar{W}_s} = \frac{\mathrm{d}I_s}{\mathrm{d}W_s} \tag{20}$$

the new averaged state of the solid ($\bar{I}_s + \Delta\bar{I}_s$, $\bar{W}_s + \Delta\bar{W}_s$) lies on the ray parallel to the previous one, which starts at ($\bar{I}_s$, $\bar{W}_s$). However, this new state should correspond with the new depth δ'. In order to find δ' it should be noted that for the variable δ' the primed equations (16) and (17) determine a curve $\bar{I}'_s = f(\bar{W}'_s)$ and δ' is the parameter of this curve. Therefore, the intersection of this curve (curve L on Fig. 3) with the ray $\mathrm{d}\bar{I}_s/\mathrm{d}W_s$ starting at ($\bar{I}_s$, $\bar{W}_s$) allows one to determine the new state ($\bar{I}_s$, $\bar{W}'_s$) and the true value δ'. With this value δ' corresponds to the new directional coefficients $\mathrm{d}I'_s/\mathrm{d}W'_s$ and $\mathrm{d}\bar{I}'_s/\mathrm{d}\bar{W}'_s$ for which the procedure is continued. The outcome of the computations are the curves $I_s(W_s)$ and $\bar{I}_s(\bar{W}_s)$ characterizing the paths of the averaged core and the averaged total solid as well as the time function $\tau(W_s)$ obtained from the integration of equation (12) in which k'_g is replaced by k'_{gb} and W_s by $\bar{W}_s$.

4. OPTIMIZATION RELATIONSHIPS (STATE EQUATIONS AND CONSTRAINTS)

Further on is sufficient to restrict our considerations to the case when the boundary region B does not exist, i.e. when the evaporation occurs on the geometrical surface of the solid (the more involved model including phase B is treated in optimization in the same manner providing that the parameters I_s and W_s, appearing in the equations below, are replaced by $\bar{I}_s$ and $\bar{W}_s$, respectively).

The peculiarity of drying optimization models consists in the fact that the so-called state equations, i.e. the independent differential equations constraining the state changes of the controlling gas phase and the controlled solid phase, must be generally implicit. This is due to the fact that the interface parameters cannot be analytically eliminated from the drying kinetics model (the consequence of the complex equilibrium relationships at the interface and the complicated nature of drying kinetics). Since, however, the numerical (or graphical) procedure for solving drying kinetics equations (10)–(12) has already been proposed it can be concluded that the solution found describes tabularly the drying kinetics in the following form

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = f(t_g, X_g, I_s, W_s) \tag{21}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = g(t_g, X_g, I_s, W_s) \tag{22}$$

(the functions f and g are the result of computations).

Optimization problems are formulated for a prescribed initial solid state ($I_{s0} = \check{I}_{s0}$, $W_{s0} = \check{W}_{s0}$) and for the preassigned, by technical requirements, final moisture content of the solid, $W_{sf} = \check{W}_{sf}$. The final solid enthalpy I_{sf} (or temperature t_{sf}) can be free or preassigned. The additional (algebraic) constraints are usually imposed on the drying gas state which has to lie within the range defined by inequalities (the asterisk corresponds with the allowable values)

$$t_{g*} \leqslant t_g \leqslant t^*_g, \quad X_g \geqslant 0, \quad \beta_g(t_g, X_g) \leqslant \beta^*_g. \tag{23}$$

Sometimes one of the quantities, t_g or X_g, is kept constant. If, for instance, t_g is constant the optimal control problem for isothermal drying will appear.

Equations (20)–(23) can constitute the model sufficient for optimization only then when the amount of gas is large in relation to the solid, e.g. when the single solid is dried by a gas with variable state ($G/S_0 \to \infty$).

The occurrence of drying in definite equipment where G/S_0 is finite and realization of the definite type of contact between gas and solid leads unavoidably to yet another constraining equation linking gas and solid states which constitute the most difficult restrictions imposed on the state and control variables. If, for example the drying occurs in nonadiabatic cocurrent or countercurrent equipment the equations which should be added to the model, equations (21)–(23), are

$$i_g = \frac{i_{m1} - I_s}{\theta} + q_g \tag{24}$$

$$X_g = \frac{X_{m1} - W_s}{\theta} + m_g \tag{25}$$

where

$$\theta = \begin{cases} +G/S & \text{for cocurrent flow where} \quad v_s > 0 \\ -G/S & \text{for countercurrent flow where} \quad v_s < 0. \end{cases} \tag{26}$$

and $q_g = Q/G$ and $m_g = \dot{M}/G$ are the heat added and moisture added (from the external surroundings) to the unit mass of dry gas, respectively

$$X_{m1} = \theta X_{g1} + W_{s1} \tag{27}$$

$$i_{m1} = \theta i_{g1} + I_{s1} \tag{28}$$

and the subscript 1 pertains to the inlet conditions for cocurrent contacting or inlet gas and outlet solid conditions for countercurrent contacting. Expressing t_g in equations (21) and (22) as a function of i_g and X_g, i.e. taking into account the set

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \tilde{f}(i_g, X_g, I_s, W_s) \tag{29}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = \tilde{g}(i_g, X_g, I_s, W_s) \tag{30}$$

one can substitute equations (24) and (25) into equations (29) and (30) obtaining

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \tilde{\tilde{f}} \times \left[q_g + \frac{i_{m1} - I_s}{\theta},\ m_g + \frac{X_{m1} - W_s}{\theta},\ I_s, W_s \right] \quad (31)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = \tilde{\tilde{g}} \times \left[q_g + \frac{i_{m1} - I_s}{\theta},\ m_g + \frac{X_{m1} - W_s}{\theta},\ I_s, W_s \right]. \quad (32)$$

Together with constraints (23) the above set may serve to optimize the cocurrent and countercurrent dryers. It may be noted that when the initial solid state is given the following process decisions can be considered: external heat q_g, external mass m_g, gas flow per unit flow of solid $|\theta|$, contact type (cocurrent or countercurrent) sign θ, inlet gas state (i_{g1}, X_{g1}), outlet solid enthalpy (or temperature) for countercurrent flow. Assuming that the process optimized must be at steady state and considering that the realization of external input of mass along the dryer is very cumbersome one may see that usually $m_g = 0$ and the only realistic control variable which can vary with solid residence time is the external heat q_g. Other decisions ($|\theta|$, sign θ, i_{g0}, X_{g0}, I_{s0}) are not time functions but numbers (parameters). Although possibilities of the non-parametric control are limited here to the function $q_g(W_s)$ the change of parametric decisions strongly effects the economical performance. Therefore, the examining of their influence on the optimal solution is also important.

An alternative formulation of the optimal control problem for the cocurrent and countercurrent dryers can arise if instead of heat q_g one considers the temperature t_h of some external phase (heating the dryer) as the possible control (Fig. 4). This is, of course, connected with expressing q_g from equation (24) in terms of t_h. Assuming that the heat exchange occurs between the external phase (e.g. hot steam) and the gaseous phase only and that α_h is the corresponding heat exchange coefficient one has for $m_g = 0$

$$q_g = \int_{\tau_1}^{\tau} \frac{v_s \alpha_h a_v}{\theta S} [t_h - t_g(i_g, X_g(W_s))]\,\mathrm{d}\tau \quad (33)$$

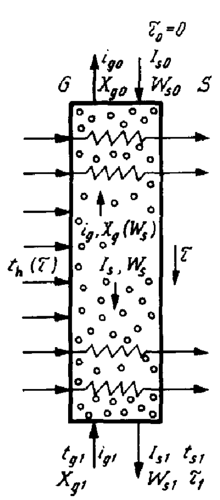

FIG. 4. Nonadiabatic control countercurrent dryer for dispersed solids.

where a_v is the heating surface area per unit volume of the dryer. Equations (24) and (33) taken simultaneously constitute an integral relationship showing that, in general, the nonadiabacity constraint should be rather of nonlocal nature as the heat q_g, equation (33), depends not only on the present state of the solid (I_s, W_s) and gas (i_g) but also on the entire past history of gas in the dryer. Therefore, in a more exact treatment, the control q_g[tacitly assumed in equation (24) as a function $q_g(W_s)$ and not a functional] should be replaced by the more proper control $t_h(W_s)$. The set of equations (24) and (33) can be transformed to a single differential relationship of the form

$$\frac{\mathrm{d}i_g}{\mathrm{d}\tau} = -\frac{1}{\theta}\frac{\mathrm{d}I_s}{\mathrm{d}\tau} + \frac{v_s \alpha_h a_v}{\theta S}[t_h - t_g(i_g, X_g(W_s))], \quad (34)$$

which represents the additional state equation to be added to the set, equations (2) and (4). Taking again W_s as the independent variable one obtains from equations (31), (32) and (34)

$$\frac{\mathrm{d}i_g}{\mathrm{d}W_s} = -\frac{\tilde{f}(I_s, W_s, i_g)}{\theta} + \frac{v_s \alpha_h a_v}{\theta S}(t_h - t_g)\tilde{g}(I_s, W_s, i_g). \quad (35)$$

Equations (31), (32) and (35) constitute the complete set of the state equations for cocurrent and countercurrent dryers.

The functions $\tilde{f}$ and $\tilde{g}$ appearing in equation (35) characterize the internal gas–solid exchange. They have already been determined by the numerical procedure described in Section 3 and they play a key role in every optimization model (state equations) of any drying equipment.

Another objective of the modelling is the nonadiabatic crosscurrent process of fluidization drying in a horizontal exchanger (Fig. 5). Assuming perfect mixing of gas and solid in a vertical plane as well as $m_g = 0$ one arrives at the following set of balance and

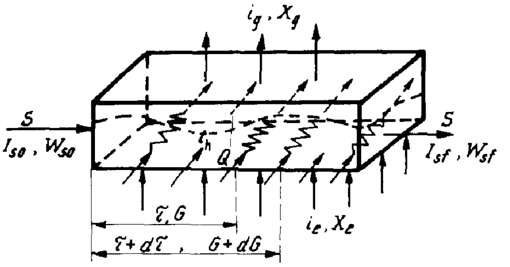

FIG. 5. Drying in a horizontal fluidized exchanger heated externally by steam.

kinetic equations

$$\frac{\mathrm{d}I_s}{\mathrm{d}\tau} = \frac{g'}{S}(i_e - i_g) + \frac{V_s \alpha_h a_v}{S}(t_h - t_g(i_g, X_g))$$

$$= \tilde{f}\tilde{g}^{-1}(i_g, X_g, I_s, W_s) \quad (36)$$

$$\frac{\mathrm{d}W_s}{\mathrm{d}\tau} = \frac{g'}{S}(X_e - X_g) = \tilde{g}^{-1}(i_g, X_g, I_s, W_s). \quad (37)$$

Here $g' = \mathrm{d}G/\mathrm{d}\tau$ is the gas flow per unit residence time of solid. Again equations (29) and (30) have been exploited. The controls are the external (inlet) gas parameters (i_e, X_e) and the temperature of heating steam t_h.

When W_s is the independent variable the above set can be transformed into the form

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \frac{i_e - i_g}{X_e - X_g} + \frac{C_h(t_h - t_g(i_g, X_g))}{X_e - X_g}$$

$$= \tilde{f}(i_g, X_g, I_s, W_s) \quad (38)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = \frac{1}{g'S^{-1}(X_e - X_g)} = \tilde{g}(i_g, X_g, I_s, W_s);$$

$$\times\left(C_h \equiv \frac{v_s \alpha_h a_v}{g}\right). \quad (39)$$

However, the above set does not yet represent the state equations that are usually applied in optimization because the set contains not only the state variables (I_s, W_s) and controls (i_e, X_e, t_h) but also additional variables (i_g, X_g) which describe the outlet (or internal) gas state. Thus, it is necessary to eliminate variables (i_g, X_g) from the set equations (38) and (39). In general such an elimination can be realized only numerically with the help of the trial and error method. In this method the pair of values (i_g, X_g) is assumed and verified for the definite values of I_s, W_s, i_e, X_e and t_h in order to obtain the same values of $\mathrm{d}I_s/\mathrm{d}W_s$ and $\mathrm{d}\tau/\mathrm{d}W_s$ for every term of equations (38) and (39). At the same time the values $\tilde{f}$ and $\tilde{g}$ have to be computed by the procedure outlined in Section 3. Thus the verification procedure is vs both interface and internal gas parameters. Although it is a cumbersome procedure it is realizable.

In some cases when the functions $\tilde{f}$ and $\tilde{g}$ are given in an analytical form, the elimination of i_g and X_g can be performed analytically and the analytical (explicit) form of the state equations is obtained. To illustrate the procedure consider as an example the case when $Le = 1$ and the transfer resistances of solid are negligible ($Bi_q \to 0$, $Bi_m \to 0$, $t_i \to t_s$, $X_i \to X_s$, etc.). Then equations (10)–(12) lead to the following expressions

$$\tilde{f} = \frac{i_g - i_s(I_s, W_s)}{X_g - X_s(I_s, W_s)}, \quad (40)$$

$$\tilde{g} = \frac{1}{k'_g a_s(X_g - X_s(I_s, W_s))}. \quad (41)$$

Eliminating X_g from equations (41) and (39) yields

$$X_g = \frac{(k'_g a_s/g'S^{-1})X_s + X_e}{(k_g a_s/g'S^{-1}) + 1}. \quad (42)$$

Hence one can determine the ratio

$$\frac{X_e - X_g}{X_g - X_s} = \frac{k'_g a_s}{g'S^{-1}}. \quad (43)$$

The above result makes it possible to obtain the following from equations (38) and (40)

$$\frac{i_e - i_g}{i_g - i_s} + \frac{C_h[t_h - t_g(i_g, X_g)]}{i_g - i_s} = \frac{k'_g a_s}{g'S^{-1}}. \quad (44)$$

Using the enthalpy formula

$$[i_g = C_g(t_g - 273.2) + r_0 X_g]$$

$$(C_g = c_g + X_g c_w; r_0 = 2502\,\mathrm{kJ\,kg^{-1}}) \quad (45)$$

to define function $t_g(i_g, X_g)$ one can eliminate i_g from equation (44) obtaining

$$i_g = \left(\frac{k'_g a_s}{g'S^{-1}} i_s + i_e + \frac{C_h}{C_g}[C_g(t_h - 273.2)\right.$$

$$\left. + r_0 X_g(X_e, X_s)]\right) \Big/ \left(\frac{k'_g a_s}{g'S^{-1}} + 1 + \frac{C_h}{C_g}\right) \quad (46)$$

where $X_g(X_e, X_s)$ is given by equation (42). The quantities X_g and i_g, equations (42) and (46), respectively, can now be substituted into expressions (40) and (41). This procedure transforms equations (38) and (39) into the form

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \frac{\left(\frac{k'_g a_s}{g'S^{-1}} + 1\right)\left\{i_e - i_s(I_s, W_s) + \frac{C_h}{C_g}[C_g(t_h - 273.2) + r_0 X_g(X_e, X_s) - i_s(I_s, W_s)]\right\}}{\left(\frac{k'_g a_s}{g'S^{-1}} + 1 + \frac{C_h}{C_g}\right)[X_e - X_s(I_s, W_s)]}, \quad (47)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}W_s} = \left(\frac{1}{g'S^{-1}} + \frac{1}{k'_g a_s}\right)\frac{1}{X_e - X_s(I_s, W_s)}. \quad (48)$$

For the given analytical functions $X_s(I_s, W_s)$ and $i_s(I_s, W_s)$, see refs. [9, 12], the above equations constitute the analytical state equations of the horizontal exchanger with the controls i_e, X_e and t_h. Other examples of the elimination procedure are given elsewhere [12]. Equations (47) and (48) are valid for the situation when the outlet gas is not in equilibrium with the solid and, therefore, the form of these equations depends on the specific form of the kinetic functions $\tilde{f}$ and $\tilde{g}$, equations (40) and (41) for $Le = 1$. However, in the industrial drying apparatus, where the height of the bed is of the order of 0.5 m, the quantity $(k'_g a_s/g'S^{-1})$ (or $(k'_g a_s S_0/G)$) is very large and equations (42) and (46) imply the equilibrium situation ($X_g \cong X_s$ and $i_g \cong i_s$) this fact being known from many experiments. In this situation the state equations (47) and (48) simplify to

$$\frac{\mathrm{d}I_s}{\mathrm{d}W_s} = \frac{i_e - i_s(I_s, W_s) + C_h(t_h - t_s(I_s, W_s))}{X_e - X_s(I_s, W_s)} \quad (49)$$

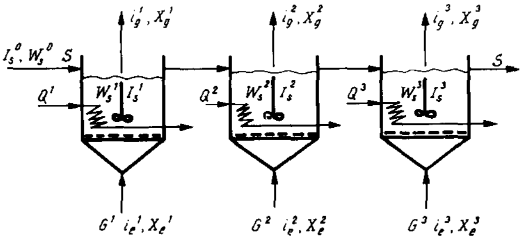

FIG. 6. Scheme of the crosscurrent nonadiabatic cascade of fluidized dryers.

$$\frac{d\tau}{dW_s} = \frac{S_0}{G} \frac{1}{X_e - X_s(I_s, W_s)} \tag{50}$$

(the equality $S/g' = S_0/G$ is exploited). These equations are independent of the interphase transfer quantities and, therefore, they are not affected by the specific forms of $\tilde{f}$ and $\tilde{g}$ and hence by the values of *Le* and *Lu*. Therefore, the only condition for equations (49) and (50) to be valid is the equilibrium condition. Indeed, taking $i_g = i_s$, $X_g = X_s$ and $t_g = t_s$ in equations (38) and (39) equations (49) and (50) are obtained without any additional assumptions.

Although the processes, considered so far, are continuous by nature, in order to make use of computer facilities, the state equations should be written in discrete (i.e. difference) form. When the discretization is performed in such a manner that the discrete unit can resemble a stage with an ideal mixing, the continuous process is replaced by a multistage cascade. For example, the suitable discretization of the model of horizontal exchanger, equations (49) and (50), yields the following equations of the crosscurrent nonadiabatic cascade of fluidized dryers (Fig. 6)

$$\frac{I_s^n - I_s^{n-1}}{W_s^n - W_s^{n-1}} = \frac{i_e^n - i_s(I_s^n, W_s^n) + C_h^n(t_h^n - t_s(I_s^n, W_s^n))}{X_e^n - X_s(I_s^n, W_s^n)} \tag{51}$$

$$\frac{\phi^n - \phi^{n-1}}{W_s^n - W_s^{n-1}} = \frac{1}{X_e^n - X_s(I_s^n, W_s^n)} \tag{52}$$

(the dimensionless time $\phi = G\tau/S_0 = G/S$ is introduced). The adiabatic counterpart of the above model written in the form

$$I_s^n - I_s^{n-1} = \theta^n[i_e^n - i_s(I_s^n, W_s^n)] \tag{53}$$

$$W_s^n - W_s^{n-1} = \theta^n[X_e^n - X_s(I_s^n, W_s^n)] \tag{54}$$

$$\phi^n - \phi^{n-1} = \theta^n \tag{55}$$

is optimized in a separate paper [13] by a special discrete maximum principle with a constant Hamiltonian [3] for the case involving solid recirculation.

5. THE OPTIMIZATION CRITERIA

In principle, there are two basic criteria of optimization for various drying processes. The first one is the drying time criterion, τ_f, and the second is the total drying cost criterion, K, involving the sum of the operation and investment costs. The τ_f criterion is one of the easiest as it is independent of the economic data. However, its applicability is restricted to those cases when the changes of the operational costs (such as drying agent cost, heat cost, pumping cost, etc.) can be neglected in comparison with the changes of the investment costs. In fact, the minimum time problem is a type of the simplest problem of optimal design. It becomes especially appropriate when the investment cost of the dryer is linear vs the drying time.

If the conditions mentioned above do not hold, the general criterion of the sum of the operational and investment costs

$$K = K^i + K^e \tag{56}$$

should be accepted. However, in concrete applications, the variable parts of these costs rather than the costs themselves are taken into consideration, these parts being dependent on the process path. The appropriateness of such an approach is due to the fact that the presence of any constant terms in K has no influence on the optimal strategy.

Consequently K^e is taken as the economic value of all energetic inputs, i.e. it describes the total economic value of heat (Q), work (L) and inlet substances ($\dot{M}_j$)

$$K^e = \left(\tilde{C}_q Q + \tilde{C}_L L + \sum_{j=1}^{k} \tilde{C}_{sj} \dot{M}_j\right) S^{-1}. \tag{57}$$

The prices $\tilde{C}_y$, $\tilde{C}_L$ and $\tilde{C}_{sj}$ are generally considered as functions of the adequate thermodynamic state parameters (e.g. $\tilde{C}_q$ depends on the temperature of heating steam, etc.).

Forms of these functions can be determined with the use of the exergy (available energy) tariff [3]. More details about the exergy approach can be found in refs. [12, 14]. Of course another scale price can be alternatively used at a particular factory.

The unit investment costs, K^i, are related to the total economic value of dryer J

$$K^i = \left(\frac{\bar{z}}{T_{gr}} + \beta\right) T_u^{-1} J S^{-1}. \tag{58}$$

For the optimal control problems the processes in a definite dryer are considered and J, τ_f and K^i are constants. If the dryer utilization time, T_u, does not vary the minimization of costs K^e can be performed. Whereas for more difficult problems of the optimal design, where J, τ_f and K^i vary, the total costs, K, should be minimized. The generalization of the K criterion for the variable amount and quality of drying products is discussed elsewhere [3].

Equations (56)–(58) are quite general and, as such, they define the performance index, K, for the majority of dryers of various scales and types. However, in a concrete optimization problem K must be expressed in terms of optimization variables, i.e. state and control variables. Such an expression must constitute an explicit analytical formula (describing some function or functional) appearing in the optimization model together with the constraints. An example of such a formula for fluidized drying costs is given in refs. [6, 13]. In this work a noneconomic performance criterion is investigated in detail exclusively, considering the rigorous formulation and solution of a minimum drying time problem together with an outline of its properties.

6. A MINIMUM TIME PROBLEM

From equation (50) describing $d\tau/dW_s$ for the equilibrium fluidized exchanger the total drying time, τ_f, can be found as the functional

$$\tau_f = \tilde{\tau}\int_{W_{s0}}^{W_{sf}} \frac{dW_s}{X_e - X_s(I_s, W_s)} \quad \left(\tilde{\tau} \equiv \frac{S_0}{G}\right). \tag{59}$$

In general, this integral depends on the path $I_s(W_s)$ and, therefore, the following optimization problem can be stated: minimize τ_f, equation (59), subject to the differential constraint, equation (49), by the suitable choice of the temperature of heating (or cooling) medium, t_h, such that

$$t_* \leqslant t_h(W_s) \leqslant t^*. \tag{60}$$

It is assumed that the parameters of the inlet gas state (i_e, X_e) are the given constants obeying the inequalities of the type (23). The initial state of solid (I_{s0}, W_{s0}) and the final moisture content W_{sf} are prescribed, whereas the final solid enthalpy I_f is free or prescribed.

A brief presentation of an exact method of solving the problem is given. Casting the problem into the format of Pontriagin's principle we introduce the Hamiltonian

$$H = z\frac{i_e - i_s(I_s, W_s) + C_h(t_h - t_s(I_s, W_s))}{X_e - X_s(I_s, W_s)} + \frac{S_0}{G(X_e - X_s(I_s, W_s))} \tag{61}$$

in which the adjoint variable z obeys the differential equation

$$\frac{dz}{dW_s} = -\frac{\partial H(I_s, W_s, t_h)}{\partial I_s}. \tag{62}$$

For the free final enthalpy I_{sf} the boundary condition holds

$$z(W_{sf}) = 0 \tag{63}$$

and hence the final value of derivative (62) becomes

$$\left(\frac{dz}{dW_s}\right)_f = -\frac{S_0\,\partial X_s/\partial I_s}{G(X_e - X_s)^2}. \tag{64}$$

As the equilibrium humidity, X_s, increases with solid temperature (or with solid enthalpy taken at constant W_s) the inequality $(\partial X_s/\partial I_s)_w > 0$ holds and hence

$$\left(\frac{dz}{dW_s}\right)_f < 0 \tag{65}$$

which implies together with equation (63) that $z < 0$ at least for some final part of the process.

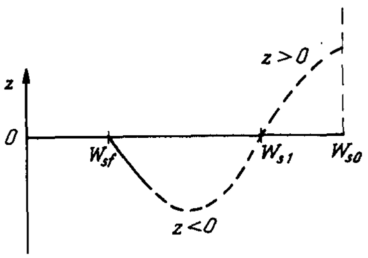

FIG. 7. If W_{s1} exists then dz/dW_s must be positive at W_{s1}.

According to Pontriagin's principle the optimal control, i.e. the control t_h which minimizes the time, equation (59), makes the Hamiltonian (61) minimal at constant z, I_s and W_s. For $z < 0$ and constant positive C the minimum H occurs when t_h attains the highest allowable value t^*. Now assume that z can change its sign within the range (W_{sf}, W_{s0}), which would imply the switching of the optimal policy from t^* to t_* at some interior point W_{s1} where $z = 0$. It is easy to conclude, see Fig. 7, that at such a point

$$\left(\frac{dz}{dW_s}\right)_1 > 0. \tag{66}$$

However, if $z = 0$ the adjoint equation, implies that

$$\frac{dz}{dW_s} = -\frac{\partial H}{\partial I_s} = -\frac{S_0}{G}\frac{\partial X_s/\partial I_s}{(X_e - X_s)^2} < 0 \tag{67}$$

for arbitrary W_s. Therefore, the switching point W_{s1} does not exist, i.e. $z < 0$ for arbitrary W_s in the range (W_{sf}, W_{s0}). It means that the optimal control is

$$t_h(W_s) = t^* \qquad W_{sf} \leqslant W_s \leqslant W_{s0} \tag{68}$$

i.e. in the whole range of W_s the temperature of the heating medium should be as high as possible. It may be noted that this simple result cannot be generally valid when the cost of the heating medium is taken into consideration as an alternative optimization criterion. For too high t_h then there appears a natural penalty in the cost expression [12] which decreases optimal t_h.

Furthermore, it can be proved that the simple control (68) must be replaced by a somewhat more complex one when the final solid enthalpy becomes prescribed and its value is sufficiently low. This is associated with the technical requirement of a sufficiently low final temperature of solid.

In this case $z(W_{sf})$ is not known, i.e. condition (63) does not hold. However, from the dynamic programming derivation of the Pontriagin's principle equations one knows that

$$z = -\frac{\partial \tau_f(I_{sf}, W_{sf})}{\partial I_{sf}} \qquad (\tau_0 \equiv 0). \tag{69}$$

Since τ_f attains the minimum vs I_{sf} at the point where $I_{sf} = I_{sfree}$ (then $z = 0$ and the previous results are valid) it can be concluded that

$$z(W_{sf}) > 0 \quad \text{for} \quad I_{sf} < I_{sfree}. \tag{70}$$

Therefore, for sufficiently low final enthalpies of solid a final part of the optimal control exists for which

$$t_h(W_s) = t_* W_{sf} \leqslant W_s \leqslant W_{s1} < W_{s0} \qquad (71)$$

For this part the inequality $z > 0$ and the adjoint equation (62) imply that

$$\frac{dz}{dW_s} < 0. \qquad (72)$$

The above result was obtained after differentiation of Hamiltonian (61) vs I_s and after exploiting the positiveness of the derivatives $\partial i_s/\partial I_s$, $\partial t_s/\partial I_s$ and $\partial X_s/\partial I_s$. In general the cooling condition, $t_h < t_s$, is sufficient for inequality (72) to hold.

From conditions (70) and (72) one can conclude that z must decrease when W_s increases until the switching moisture content W_{s1} is attained for which $z = 0$. Next, the part appears where $z < 0$ and for which $t_h = t^*$. Thus in order to achieve a sufficiently low temperature of the final solid a two-part operation is required. The initial part of the process is the proper drying in which the heating medium is used at the highest allowable temperature t^*. The final part is essentially the cooling of the dryer in which the changes of the moisture content W_s become relatively small and the changes of the solid temperature t_s are considerable. In this part the lowest allowable temperature of the cooling medium, t_*, is used. The switching solid state (I_{s1}, W_{s1}), at which t_h changes from t^* into t_*, can be found by solving numerically the canonical set, equations (49) and (62), with the boundary conditions imposed on I_{s0}, W_{s0}, I_{sf} and W_{sf} (two-point boundary value problem). It can be noted that for very low I_{sf} only t_* is optimal.

It is worth noting that all the qualitative results discussed above have been found without taking into account any special properties of the solid. These results establish a rigorous theoretical justification of the observations known occasionally from drying practices which have presently been confirmed by a rigorous optimization analysis. A more involved optimization treatment including the economical optimization criterion, K, should be a subject of further effort.

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MODELISATION A PARAMETRES LOCALISES—UNE INTRODUCTION A L'OPTIMISATION DES SYSTEMES A SECHAGE MONODIMENSIONNEL ET NON ADIABATIQUE

Résumé—On propose un modèle à paramètre localisé, résolu numériquement pour les systèmes à séchage monodimensionnel. Le modèle est une amélioration de la description classique du séchage, basée sur le coefficient de séchage. On prend en considération les changements d'enthalpie du solide, les résistances partielles au transfert de chaleur et de masse dans les phases gazeuse et solide, et les variations des paramètres d'interface. Une description de la cinétique du séchage est donnée pour s'appliquer à toutes les périodes du séchage et elle clarifie l'établissement de l'état de bulbe humide. On porte une attention à la cinétique du séchage avec déplacement du front d'évaporation. Des modèles d'optimisation de plusieurs séchoirs (co-courant, contre-courant et type croisé) sont dérivés de la cinétique et des équations de bilan. Les modèles contiennent des contraintes différentielles, comme les équations d'état, des contraintes algébriques et des indices de performance (critère d'optimisation) comme le temps de séchage ou le coût financier total. Le problème du temps minimal de séchage est résolu pour un échangeur horizontal à fluidisation et on montre une politique optimale pour les températures finales d'un solide. D'autres résultats d'optimisation seront présentés pour les modèles des procédés restants.

ENTWICKLUNG EINES ALLGEMEINEN PARAMETERMODELLES UND EINFÜHRUNG IN DIE OPTIMIERUNG FÜR EINDIMENSIONALE NICHTADIABATE TROCKNUNGSSYSTEME

Zusammenfassung—Ein allgemeines Parametermodell für die Trocknung in verschiedenen eindimensionalen Systemen wird vorgeschlagen. Die Lösung erfolgt numerisch. Das Modell ist eine Erweiterung der üblichen Beschreibung von Trocknungsvorgängen, die auf dem Trocknungskoeffizienten beruhen. Die Erweiterung berücksichtigt Enthalpieänderungen im festen Zustand, Wärme- und Stofftransportwiderstände in der festen und der Gasphase und die Änderung aller Grenzflächen-Parameter. Eine einheitliche Beschreibung der Trocknungskinetik wird angegeben, die auf alle Phasen der Trocknung anwendbar ist, und die unter anderem die Entwicklung des Feuchtzustandes erklärt. Einige Beachtung wird der Trocknungskinetik mit einer wandernden Verdampfungsfront gewidmet. Optimierungsmodelle für verschiedene Trockner (Gleich-, Gegen- und Kreuzstrom) werden auf der Grundlage der kinetischen und der Bilanzgleichungen hergeleitet. Die Modelle enthalten Differential- oder Differenzenbedingungen, d.h. Zustandsgleichungen, algebraische Bedingungen und außerdem einen Güteindex (Optimierungskriterium), der entweder die Trockenzeit oder die Gesamtkosten enthält. Das Problem der minimalen Trocknungszeit wird für einen in einem waagerechten Fließbettaustauscher ablaufenden Prozeß gelöst, sowie das optimale Vorgehen für niedrige Feststoff-Endtemperaturen aufgezeigt. Optimierungsergebnisse für die Modelle der übrigen Systeme werden später berichtet werden.

КУСОЧНО-ПАРАМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ И ПОДХОД К ПРОБЛЕМЕ ОПТИМИЗАЦИИ ОДНОМЕРНЫХ НЕАДИАБАТИЧЕСКИХ СИСТЕМ

Аннотация—Предложена кусочно-параметрическая модель процесса сушки в некоторых одномерных системах, для которой можно получить численное решение и которая является усовершенствованным вариантом стандартного метода описания процесса на основе коэффициента сушки. В модели учитывается изменение энтальпии твердой фазы, парциальные сопротивления переносу тепла и массы в газовой и твердой фазах и изменение всех параметров на границе раздела фаз. Дано унифицированное описание кинетики сушки, которое можно применить ко всем ее стадиям и которое, помимо всего прочего, учитывает установление состояния мокрого термометра. Рассмотрена также кинетика сушки при движении фронта испарения. На основе кинетических и балансных уравнений построены модели оптимизации ряда сушилок (со спутным, противоточным и перекрестным течениями). Модели включают дифференциальные или разностные связи, т.е. уравнения состояния, алгебраические связи, а также показатель качества работы сушилок (критерий оптимизации), представленный через время сущки или суммарную экономическую стоимость. Решена проблема минимизации времени сушки для теплообменника с горизонтальным псевдоожиженным слоем и показан нетривиальный характер оптимального режима в случае низких конечных температур твердого материала. Приведены также другие результаты по оптимизации, полученные на моделях других процессов.